

3.5 Solving Two-Step Equations

Essential Question How can you use algebra tiles to solve a two-step equation?

1 ACTIVITY: Solving a Two-Step Equation

Work with a partner. Use algebra tiles to model and solve $2x - 3 = -5$.

Model the equation $2x - 3 = -5$.



Remove the red tiles on the left side by adding yellow tiles to each side.



How many zero pairs can you remove from each side? Circle them.



Because there are green tiles, divide the red tiles into equal groups. Circle the groups.



Keep one of the groups. This shows the value of x .



So, $x =$.

2 ACTIVITY: The Math behind the Tiles

Work with a partner. Solve $2x - 3 = -5$ without using algebra tiles. Complete each step. Then answer the questions.

Use the steps in Activity 1 as a guide.

$$2x - 3 = -5$$

Write the equation.

$$2x - 3 + \text{} = -5 + \text{}$$

Add to each side.

$$2x = \text{}$$

Simplify.

$$\frac{2x}{\text{}} = \frac{\text{}}{\text{}}$$

Divide each side by .

$$x = \text{}$$

Simplify.

So, $x =$.

- Which step is first, adding 3 to each side or dividing each side by 2?
- How are the above steps related to the steps in Activity 1?

Solving Equations

In this lesson, you will

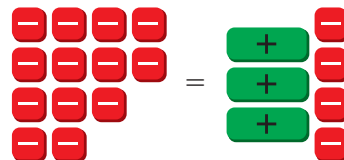
- solve two-step equations.
- solve real-life problems.

3 ACTIVITY: Solving Equations Using Algebra Tiles

Work with a partner.

- Write an equation shown by the algebra tiles.
- Use algebra tiles to model and solve the equation.
- Check your answer by solving the equation without using algebra tiles.

a. 

b. 

4 ACTIVITY: Working Backwards

Work with a partner.

- a. **Sample:** Your friend pauses a video game to get a drink. You continue the game. You double the score by saving a princess. Then you lose 75 points because you do not collect the treasure. You finish the game with -25 points. How many points did you have when you started?

One way to solve the problem is to work backwards. To do this, start with the end result and retrace the events.

You have -25 points at the end of the game. -25

You lost 75 points for not collecting the treasure, so add 75 to -25 . $-25 + 75 = 50$

You doubled your score for saving the princess, so find half of 50. $50 \div 2 = 25$

❖ So, you started the game with 25 points.

- b. You triple your account balance by making a deposit. Then you withdraw \$127.32 to buy groceries. Your account is now overdrawn by \$10.56. By working backwards, find your account balance before you made the deposit.

Math Practice

Maintain Oversight

How does working backwards help you decide which operation to do first? Explain.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use algebra tiles to solve a two-step equation?
6. When solving the equation $4x + 1 = -11$, what is the first step?
7. **REPEATED REASONING** Solve the equation $2x - 75 = -25$. How do your steps compare with the strategy of working backwards in Activity 4?

Practice

Use what you learned about solving two-step equations to complete Exercises 6–11 on page 112.

EXAMPLE 1 Solving a Two-Step Equation

Solve $-3x + 5 = 2$. Check your solution.

$$-3x + 5 = 2 \quad \text{Write the equation.}$$

Undo the addition. $\rightarrow -5 \quad -5$ Subtraction Property of Equality

$$-3x = -3 \quad \text{Simplify.}$$

Undo the multiplication. $\rightarrow \frac{-3x}{-3} = \frac{-3}{-3}$ Division Property of Equality

$$x = 1 \quad \text{Simplify.}$$

Check

$$-3x + 5 = 2$$

$$-3(1) + 5 \stackrel{?}{=} 2$$

$$-3 + 5 \stackrel{?}{=} 2$$

$$2 = 2 \quad \checkmark$$

∴ The solution is $x = 1$.

On Your Own

Solve the equation. Check your solution.

Now You're Ready
Exercises 6–17

1. $2x + 12 = 4$

2. $-5c + 9 = -16$

3. $3(x - 4) = 9$

EXAMPLE 2 Solving a Two-Step Equation

Solve $\frac{x}{8} - \frac{1}{2} = -\frac{7}{2}$. Check your solution.

$$\frac{x}{8} - \frac{1}{2} = -\frac{7}{2} \quad \text{Write the equation.}$$

$$+ \frac{1}{2} \quad + \frac{1}{2} \quad \text{Addition Property of Equality}$$

$$\frac{x}{8} = -3 \quad \text{Simplify.}$$

$$8 \cdot \frac{x}{8} = 8 \cdot (-3) \quad \text{Multiplication Property of Equality}$$

$$x = -24 \quad \text{Simplify.}$$

Check

$$\frac{x}{8} - \frac{1}{2} = -\frac{7}{2}$$

$$\frac{-24}{8} - \frac{1}{2} \stackrel{?}{=} -\frac{7}{2}$$

$$-3 - \frac{1}{2} \stackrel{?}{=} -\frac{7}{2}$$

$$-\frac{7}{2} = -\frac{7}{2} \quad \checkmark$$

∴ The solution is $x = -24$.

On Your Own

Solve the equation. Check your solution.

Now You're Ready
Exercises 20–25

4. $\frac{m}{2} + 6 = 10$

5. $-\frac{z}{3} + 5 = 9$

6. $\frac{2}{5} + 4a = -\frac{6}{5}$

Study Tip

You can simplify the equation in Example 2 before solving. Multiply each side by the LCD of the fractions, 8.

$$\frac{x}{8} - \frac{1}{2} = -\frac{7}{2}$$

$$x - 4 = -28$$

$$x = -24$$

EXAMPLE 3 Combining Like Terms Before Solving

Solve $3y - 8y = 25$.

$$3y - 8y = 25$$

Write the equation.

$$-5y = 25$$

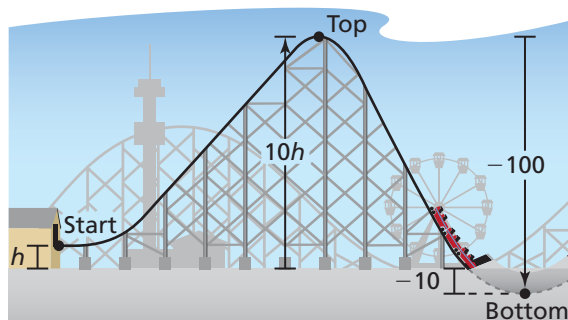
Combine like terms.

$$y = -5$$

Divide each side by -5 .

∴ The solution is $y = -5$.

EXAMPLE 4 Real-Life Application



The height at the top of a roller coaster hill is 10 times the height h of the starting point. The height decreases 100 feet from the top to the bottom of the hill. The height at the bottom of the hill is -10 feet. Find h .

Location	Verbal Description	Expression
Start	The height at the start is h .	h
Top of hill	The height at the top of the hill is 10 times the starting height h .	$10h$
Bottom of hill	The height decreases by 100 feet. So, subtract 100.	$10h - 100$

The height at the bottom of the hill is -10 feet. Solve $10h - 100 = -10$ to find h .

$$10h - 100 = -10$$

Write equation.

$$10h = 90$$

Add 100 to each side.

$$h = 9$$

Divide each side by 10.

∴ So, the height at the start is 9 feet.

On Your Own

Solve the equation. Check your solution.

7. $4 - 2y + 3 = -9$

8. $7x - 10x = 15$

9. $-8 = 1.3m - 2.1m$

10. **WHAT IF?** In Example 4, the height at the bottom of the hill is -5 feet. Find the height h .

Now You're Ready
Exercises 29–34

Vocabulary and Concept Check

1. **WRITING** How do you solve two-step equations?

Match the equation with the first step to solve it.

2. $4 + 4n = -12$ 3. $4n = -12$ 4. $\frac{n}{4} = -12$ 5. $\frac{n}{4} - 4 = -12$


A. Add 4. B. Subtract 4. C. Multiply by 4. D. Divide by 4.


Practice and Problem Solving

Solve the equation. Check your solution.

- 1 6. $2v + 7 = 3$ 7. $4b + 3 = -9$ 8. $17 = 5k - 2$
 9. $-6t - 7 = 17$ 10. $8n + 16.2 = 1.6$ 11. $-5g + 2.3 = -18.8$
 12. $2t - 5 = -10$ 13. $-4p + 9 = -5$ 14. $11 = -5x - 2$
 15. $4 + 2.2h = -3.7$ 16. $-4.8f + 6.4 = -8.48$ 17. $7.3y - 5.18 = -51.9$

ERROR ANALYSIS Describe and correct the error in finding the solution.

18. 
$$\begin{aligned} -6 + 2x &= -10 \\ -6 + \frac{2x}{2} &= \frac{-10}{2} \\ -6 + x &= -5 \\ x &= 1 \end{aligned}$$

19. 
$$\begin{aligned} -3x + 2 &= -7 \\ -3x &= -9 \\ \frac{-3x}{-3} &= \frac{-9}{-3} \\ x &= -3 \end{aligned}$$

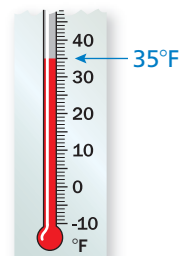
Solve the equation. Check your solution.

- 2 20. $\frac{3}{5}g - \frac{1}{3} = -\frac{10}{3}$ 21. $\frac{a}{4} - \frac{5}{6} = -\frac{1}{2}$ 22. $-\frac{1}{3} + 2z = -\frac{5}{6}$
 23. $2 - \frac{b}{3} = -\frac{5}{2}$ 24. $-\frac{2}{3}x + \frac{3}{7} = \frac{1}{2}$ 25. $-\frac{9}{4}v + \frac{4}{5} = \frac{7}{8}$

In Exercises 26–28, write an equation. Then solve.

26. **WEATHER** Starting at 1:00 P.M., the temperature changes -4 degrees per hour. How long will it take to reach -1° ?
 27. **BOWLING** It costs \$2.50 to rent bowling shoes. Each game costs \$2.25. You have \$9.25. How many games can you bowl?
 28. **CELL PHONES** A cell phone company charges a monthly fee plus \$0.25 for each text message. The monthly fee is \$30.00 and you owe \$59.50. How many text messages did you have?

Temperature at 1:00 P.M.



Solve the equation. Check your solution.

3 29. $3v - 9v = 30$

30. $12t - 8t = -52$

31. $-8d - 5d + 7d = 72$

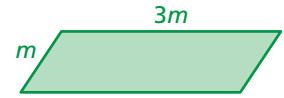
32. $6(x - 2) = -18$

33. $-4(m + 3) = 24$

34. $-8(y + 9) = -40$

35. **WRITING** Write a real-world problem that can be modeled by $\frac{1}{2}x - 2 = 8$. Then solve the equation.

36. **GEOMETRY** The perimeter of the parallelogram is 102 feet. Find m .



REASONING Exercises 37 and 38 are missing information. Tell what information you need to solve the problem.

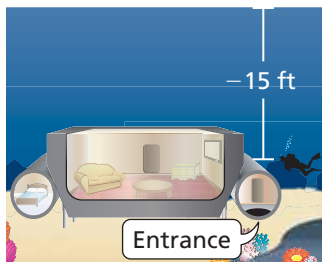
37. **TAXI** A taxi service charges an initial fee plus \$1.80 per mile. How far can you travel for \$12?

38. **EARTH** The coldest surface temperature on the Moon is 57 degrees colder than twice the coldest surface temperature on Earth. What is the coldest surface temperature on Earth?

39. **PROBLEM SOLVING** On Saturday, you catch insects for your science class. Five of the insects escape. The remaining insects are divided into three groups to share in class. Each group has nine insects. How many insects did you catch on Saturday?

a. Solve the problem by working backwards.

b. Solve the equation $\frac{x - 5}{3} = 9$. How does the answer compare with the answer to part (a)?



40. **UNDERWATER HOTEL** You must scuba dive to the entrance of your room at Jules' Undersea Lodge in Key Largo, Florida. The diver is 1 foot deeper than $\frac{2}{3}$ of the elevation of the entrance. What is the elevation of the entrance?

41. **Geometry** How much should you change the length of the rectangle so that the perimeter is 54 centimeters? Write an equation that shows how you found your answer.



Fair Game Review What you learned in previous grades & lessons

Multiply or divide. (Section 2.4)

42. -6.2×5.6

43. $\frac{8}{3} \times \left(-2\frac{1}{2}\right)$

44. $\frac{5}{2} \div \left(-\frac{4}{5}\right)$

45. $-18.6 \div (-3)$

46. **MULTIPLE CHOICE** Which fraction is *not* equivalent to 0.75? (Skills Review Handbook)

(A) $\frac{15}{20}$

(B) $\frac{9}{12}$

(C) $\frac{6}{9}$

(D) $\frac{3}{4}$