5.1 Ratios and Rates

Essential Question How do rates help you describe real-life problems?

The Meaning of a Word Rate

When you rent snorkel gear at the beach, you should pay attention to the rental **rate**. The rental rate is in dollars per hour.

1 ACTIVITY: Finding Reasonable Rates

Work with a partner.

a. Match each description with a verbal rate.

b. Match each verbal rate with a numerical rate.

c. Give a reasonable numerical rate for each description. Then give an unreasonable rate.

Description	*Verbal Rate*	*Numerical Rate*
Your running rate in a 100-meter dash	Dollars per year	$\frac{\square \text{ in.}}{\text{yr}}$
The fertilization rate for an apple orchard	Inches per year	$\frac{\square \text{ lb}}{\text{acre}}$
The average pay rate for a professional athlete	Meters per second	$\frac{\$ \square}{\text{yr}}$
The average rainfall rate in a rain forest	Pounds per acre	$\frac{\square \text{ m}}{\text{sec}}$

Ratios and Rates

In this lesson, you will

- find ratios, rates, and unit rates.
- find ratios and rates involving ratios of fractions.

2 ACTIVITY: Simplifying Expressions That Contain Fractions

Work with a partner. Describe a situation where the given expression may apply. Show how you can rewrite each expression as a division problem. Then simplify and interpret your result.

a. $\frac{\frac{1}{2} \text{ c}}{4 \text{ fl oz}}$ **b.** $\frac{2 \text{ in.}}{\frac{3}{4} \text{ sec}}$ **c.** $\frac{\frac{3}{8} \text{ c sugar}}{\frac{3}{5} \text{ c flour}}$ **d.** $\frac{\frac{5}{6} \text{ gal}}{\frac{2}{3} \text{ sec}}$

3 ACTIVITY: Using Ratio Tables to Find Equivalent Rates

Work with a partner. A communications satellite in orbit travels about 18 miles every 4 seconds.

a. Identify the rate in this problem.

b. Recall that you can use *ratio tables* to find and organize equivalent ratios and rates. Complete the ratio table below.

Time (seconds)	4	8	12	16	20
Distance (miles)					

c. How can you use a ratio table to find the speed of the satellite in miles per minute? miles per hour?

d. How far does the satellite travel in 1 second? Solve this problem (1) by using a ratio table and (2) by evaluating a quotient.

e. How far does the satellite travel in $\frac{1}{2}$ second? Explain your steps.

4 ACTIVITY: Unit Analysis

Work with a partner. Describe a situation where the product may apply. Then find each product and list the units.

a. $10 \text{ gal} \times \frac{22 \text{ mi}}{\text{gal}}$

b. $\frac{7}{2} \text{ lb} \times \frac{\$3}{\frac{1}{2} \text{ lb}}$

c. $\frac{1}{2} \text{ sec} \times \frac{30 \text{ ft}^2}{\text{sec}}$

Math Practice

View as Components

What is the product of the numbers? What is the product of the units? Explain.

What Is Your Answer?

5. IN YOUR OWN WORDS How do rates help you describe real-life problems? Give two examples.

6. To estimate the annual salary for a given hourly pay rate, multiply by 2 and insert "000" at the end.

Sample: $10 per hour is about $20,000 per year.

a. Explain why this works. Assume the person is working 40 hours a week.

b. Estimate the annual salary for an hourly pay rate of $8 per hour.

c. You earn $1 million per month. What is your annual salary?

d. Why is the cartoon funny?

"We had someone apply for the job. He says he would like $1 million a month, but will settle for $8 an hour."

Practice Use what you discovered about ratios and rates to complete Exercises 7–10 on page 167.

5.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
ratio, *p. 164*
rate, *p. 164*
unit rate, *p. 164*
complex fraction, *p. 165*

A **ratio** is a comparison of two quantities using division.

$\frac{3}{4}$, 3 to 4, 3 : 4

A **rate** is a ratio of two quantities with different units.

$\frac{60 \text{ miles}}{2 \text{ hours}}$

A rate with a denominator of 1 is called a **unit rate**.

$\frac{30 \text{ miles}}{1 \text{ hour}}$

EXAMPLE 1 Finding Ratios and Rates

There are 45 males and 60 females in a subway car. The subway car travels 2.5 miles in 5 minutes.

a. Find the ratio of males to females.

$$\frac{\text{males}}{\text{females}} = \frac{45}{60} = \frac{3}{4}$$

∴ The ratio of males to females is $\frac{3}{4}$.

b. Find the speed of the subway car.

$$2.5 \text{ miles in } 5 \text{ minutes} = \frac{2.5 \text{ mi}}{5 \text{ min}} = \frac{2.5 \text{ mi} \div 5}{5 \text{ min} \div 5} = \frac{0.5 \text{ mi}}{1 \text{ min}}$$

∴ The speed is 0.5 mile per minute.

EXAMPLE 2 Finding a Rate from a Ratio Table

The ratio table shows the costs for different amounts of artificial turf. Find the unit rate in dollars per square foot.

	× 4	× 4	× 4	
Amount (square feet)	25	100	400	1600
Cost (dollars)	100	400	1600	6400
	× 4	× 4	× 4	

Use a ratio from the table to find the unit rate.

$$\frac{\text{cost}}{\text{amount}} = \frac{\$100}{25 \text{ ft}^2}$$ Use the first ratio in the table.

$$= \frac{\$4}{1 \text{ ft}^2}$$ Simplify.

Remember
The abbreviation ft^2 means *square feet*.

∴ So, the unit rate is $4 per square foot.

On Your Own

Now You're Ready
Exercises 11–24

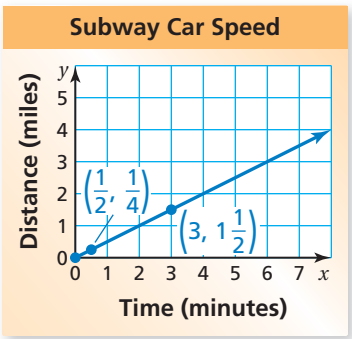
1. In Example 1, find the ratio of females to males.
2. In Example 1, find the ratio of females to total passengers.
3. The ratio table shows the distance that the *International Space Station* travels while orbiting Earth. Find the speed in miles per second.

Time (seconds)	3	6	9	12
Distance (miles)	14.4	28.8	43.2	57.6

A **complex fraction** has at least one fraction in the numerator, denominator, or both. You may need to simplify complex fractions when finding ratios and rates.

EXAMPLE 3 Finding a Rate from a Graph

The graph shows the speed of a subway car. Find the speed in miles per minute. Compare the speed to the speed of the subway car in Example 1.



Step 1: Choose and interpret a point on the line.

The point $\left(\frac{1}{2}, \frac{1}{4}\right)$ indicates that the subway car travels $\frac{1}{4}$ mile in $\frac{1}{2}$ minute.

Step 2: Find the speed.

$$\frac{\text{distance traveled}}{\text{elapsed time}} = \frac{\frac{1}{4}}{\frac{1}{2}} \quad \leftarrow \text{miles}, \leftarrow \text{minutes}$$

$$= \frac{1}{4} \div \frac{1}{2} \qquad \text{Rewrite the quotient.}$$

$$= \frac{1}{4} \cdot 2 = \frac{1}{2} \qquad \text{Simplify.}$$

∴ The speed of the subway car is $\frac{1}{2}$ mile per minute.

Because $\frac{1}{2}$ mile per minute = 0.5 mile per minute, the speeds of the two subway cars are the same.

On Your Own

Now You're Ready
Exercise 28

4. You use the point $\left(3, 1\frac{1}{2}\right)$ to find the speed of the subway car. Does your answer change? Explain your reasoning.

EXAMPLE 4 Solving a Ratio Problem

STEM VIDEO

You mix $\frac{1}{2}$ cup of yellow paint for every $\frac{3}{4}$ cup of blue paint to make 15 cups of green paint. How much yellow paint and blue paint do you use?

Method 1: The ratio of yellow paint to blue paint is $\frac{1}{2}$ to $\frac{3}{4}$. Use a ratio table to find an equivalent ratio in which the total amount of yellow paint and blue paint is 15 cups.

	Yellow (cups)	Blue (cups)	Total (cups)	
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$	
$\times 4$	2	3	5	$\times 4$
$\times 3$	6	9	15	$\times 3$

So, you use 6 cups of yellow paint and 9 cups of blue paint.

Method 2: Use the fraction of the green paint that is made from yellow paint and the fraction of the green paint that is made from blue paint. You use $\frac{1}{2}$ cup of yellow paint for every $\frac{3}{4}$ cup of blue paint, so the fraction of the green paint that is made from yellow paint is

yellow → $\frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{4}} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$.

green →

Similarly, the fraction of the green paint that is made from blue paint is

blue → $\frac{\frac{3}{4}}{\frac{1}{2} + \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{4} \cdot \frac{4}{5} = \frac{3}{5}$.

green →

So, you use $\frac{2}{5} \cdot 15 = 6$ cups of yellow paint and $\frac{3}{5} \cdot 15 = 9$ cups of blue paint.

Math Practice

Analyze Givens

What information is given in the problem? How does this help you know that the ratio table needs a "total" column? Explain.

On Your Own

Now You're Ready
Exercises 33 and 34

5. How much yellow paint and blue paint do you use to make 20 cups of green paint?

5.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** How can you tell when a rate is a unit rate?
2. **WRITING** Why do you think rates are usually written as unit rates?
3. **OPEN-ENDED** Write a real-life rate that applies to you.

Estimate the unit rate.

4. \$74.75

5. \$1.19

6. \$2.35

Practice and Problem Solving

Find the product. List the units.

7. $8 \text{ h} \times \frac{\$9}{\text{h}}$
8. $8 \text{ lb} \times \frac{\$3.50}{\text{lb}}$
9. $\frac{29}{2} \text{ sec} \times \frac{60 \text{ MB}}{\text{sec}}$
10. $\frac{3}{4} \text{ h} \times \frac{19 \text{ mi}}{\frac{1}{4} \text{ h}}$

Write the ratio as a fraction in simplest form.

11. 25 to 45
12. 63 : 28
13. 35 girls : 15 boys
14. 51 correct : 9 incorrect
15. 16 dogs to 12 cats
16. $2\frac{1}{3}$ feet : $4\frac{1}{2}$ feet

Find the unit rate.

17. 180 miles in 3 hours
18. 256 miles per 8 gallons
19. \$9.60 for 4 pounds
20. \$4.80 for 6 cans
21. 297 words in 5.5 minutes
22. $21\frac{3}{4}$ meters in $2\frac{1}{2}$ hours

Use the ratio table to find the unit rate with the specified units.

23. servings per package

Packages	3	6	9	12
Servings	13.5	27	40.5	54

24. feet per year

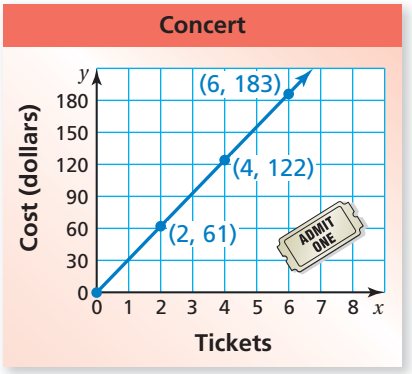
Years	2	6	10	14
Feet	7.2	21.6	36	50.4

25. **DOWNLOAD** At 1:00 P.M., you have 24 megabytes of a movie. At 1:15 P.M., you have 96 megabytes. What is the download rate in megabytes per minute?

26. POPULATION In 2007, the U.S. population was 302 million people. In 2012, it was 314 million. What was the rate of population change per year?

27. PAINTING A painter can paint 350 square feet in 1.25 hours. What is the painting rate in square feet per hour?

3 **28. TICKETS** The graph shows the cost of buying tickets to a concert.



a. What does the point (4, 122) represent?

b. What is the unit rate?

c. What is the cost of buying 10 tickets?

29. CRITICAL THINKING Are the two statements equivalent? Explain your reasoning.

- The ratio of boys to girls is 2 to 3.
- The ratio of girls to boys is 3 to 2.

30. TENNIS A sports store sells three different packs of tennis balls. Which pack is the best buy? Explain.

31. FLOORING It costs $68 for 16 square feet of flooring. How much does it cost for 12 square feet of flooring?

32. OIL SPILL An oil spill spreads 25 square meters every $\frac{1}{6}$ hour. How much area does the oil spill cover after 2 hours?

4 **33. JUICE** You mix $\frac{1}{4}$ cup of juice concentrate for every 2 cups of water to make 18 cups of juice. How much juice concentrate and water do you use?

34. LANDSCAPING A supplier sells $2\frac{1}{4}$ pounds of mulch for every $1\frac{1}{3}$ pounds of gravel. The supplier sells 172 pounds of mulch and gravel combined. How many pounds of each item does the supplier sell?

35. HEART RATE Your friend's heart beats 18 times in 15 seconds when at rest. While running, your friend's heart beats 25 times in 10 seconds.

a. Find the heart rate in beats per minute at rest and while running.

b. How many more times does your friend's heart beat in 3 minutes while running than while at rest?

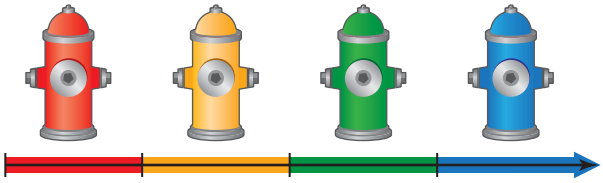
36. PRECISION The table shows nutritional information for three beverages.

Beverage	Serving Size	Calories	Sodium
Whole milk	1 c	146	98 mg
Orange juice	1 pt	210	10 mg
Apple juice	24 fl oz	351	21 mg

a. Which has the most calories per fluid ounce?

b. Which has the least sodium per fluid ounce?

37. RESEARCH Fire hydrants are painted one of four different colors to indicate the rate at which water comes from the hydrant.



a. Use the Internet to find the ranges of the rates for each color.

b. Research why a firefighter needs to know the rate at which water comes out of a hydrant.

38. PAINT You mix $\frac{2}{5}$ cup of red paint for every $\frac{1}{4}$ cup of blue paint to make $1\frac{5}{8}$ gallons of purple paint.

a. How much red paint and blue paint do you use?

b. You decide that you want to make a lighter purple paint. You make the new mixture by adding $\frac{1}{10}$ cup of white paint for every $\frac{2}{5}$ cup of red paint and $\frac{1}{4}$ cup of blue paint. How much red paint, blue paint, and white paint do you use to make $\frac{3}{8}$ gallon of lighter purple paint?

39. Critical Thinking You and a friend start hiking toward each other from opposite ends of a 17.5-mile hiking trail. You hike $\frac{2}{3}$ mile every $\frac{1}{4}$ hour. Your friend hikes $2\frac{1}{3}$ miles per hour.

a. Who hikes faster? How much faster?

b. After how many hours do you meet?

c. When you meet, who hiked farther? How much farther?

Fair Game Review What you learned in previous grades & lessons

Copy and complete the statement using <, >, or =. *(Section 2.1)*

40. $\frac{9}{2} \square \frac{8}{3}$

41. $-\frac{8}{15} \square \frac{10}{18}$

42. $\frac{-6}{24} \square \frac{-2}{8}$

43. MULTIPLE CHOICE Which fraction is greater than $-\frac{2}{3}$ and less than $-\frac{1}{2}$? *(Section 2.1)*

(A) $-\frac{3}{4}$ (B) $-\frac{7}{12}$ (C) $-\frac{5}{12}$ (D) $-\frac{3}{8}$