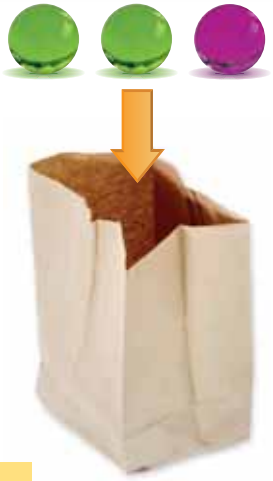
10.5 Independent and Dependent Events

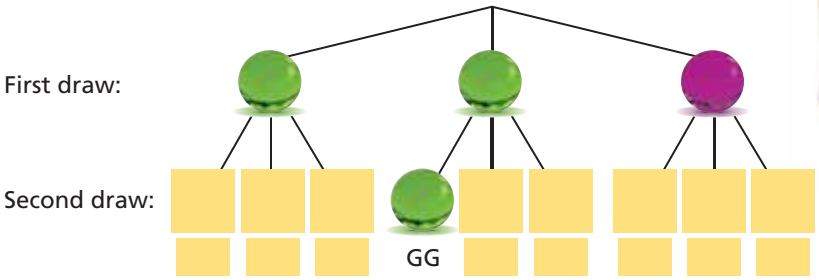
Essential Question What is the difference between dependent and independent events?

1 ACTIVITY: Drawing Marbles from a Bag (With Replacement)

Work with a partner. You have three marbles in a bag. There are two green marbles and one purple marble. Randomly draw a marble from the bag. Then put the marble back in the bag and draw a second marble.



a. Complete the tree diagram. Let G = green and P = purple. Find the probability that both marbles are green.

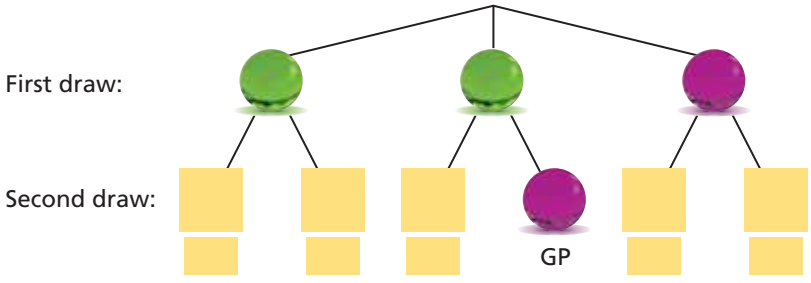


b. Does the probability of getting a green marble on the second draw *depend* on the color of the first marble? Explain.

2 ACTIVITY: Drawing Marbles from a Bag (Without Replacement)

Work with a partner. Using the same marbles from Activity 1, randomly draw two marbles from the bag.

a. Complete the tree diagram. Let G = green and P = purple. Find the probability that both marbles are green.



Is this event more likely than the event in Activity 1? Explain.

b. Does the probability of getting a green marble on the second draw *depend* on the color of the first marble? Explain.

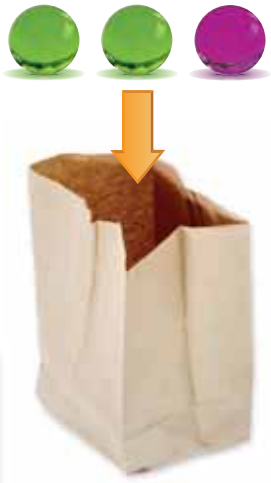
Probability and Statistics

In this lesson, you will

- identify independent and dependent events.
- use formulas to find probabilities of independent and dependent events.

3 ACTIVITY: Conducting an Experiment

Work with a partner. Conduct two experiments.



a. In the first experiment, randomly draw one marble from the bag. Put it back. Draw a second marble. Repeat this 36 times. Record each result. Make a bar graph of your results.

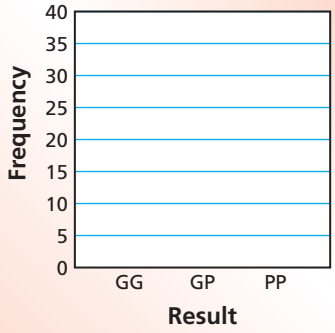
b. In the second experiment, randomly draw two marbles from the bag 36 times. Record each result. Make a bar graph of your results.

Math Practice

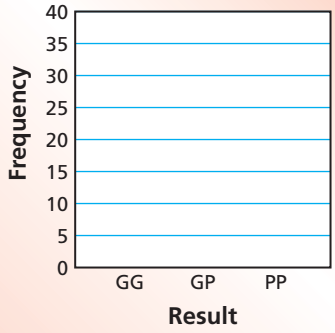
Use Definitions

In what other mathematical context have you seen the terms *independent* and *dependent*? How does knowing these definitions help you answer the questions in part (d)?

First Experiment Results



Second Experiment Results



c. For each experiment, estimate the probability of drawing two green marbles.

d. Which experiment do you think represents *dependent events*? Which represents *independent events*? Explain your reasoning.

What Is Your Answer?

4. IN YOUR OWN WORDS What is the difference between dependent and independent events? Describe a real-life example of each.

In Questions 5–7, tell whether the events are *independent* or *dependent*. Explain your reasoning.

5. You roll a 5 on a number cube and spin blue on a spinner.

6. Your teacher chooses one student to lead a group, and then chooses another student to lead another group.

7. You spin red on one spinner and green on another spinner.

8. In Activities 1 and 2, what is the probability of drawing a green marble on the first draw? on the second draw? How do you think you can use these two probabilities to find the probability of drawing two green marbles?

Practice Use what you learned about independent and dependent events to complete Exercises 3 and 4 on page 433.

10.5 Lesson

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Key Vocabulary
independent events, *p. 430*
dependent events, *p. 431*

Compound events may be *independent events* or *dependent events*. Events are **independent events** if the occurrence of one event *does not* affect the likelihood that the other event(s) will occur.

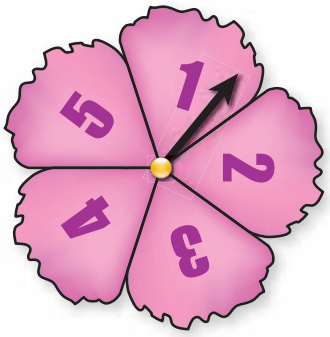
Key Idea

Probability of Independent Events

Words The probability of two or more independent events is the product of the probabilities of the events.

Symbols
$$P(A \text{ and } B) = P(A) \cdot P(B)$$
$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$

EXAMPLE 1 Finding the Probability of Independent Events



You spin the spinner and flip the coin. What is the probability of spinning a prime number and flipping tails?

The outcome of spinning the spinner does not affect the outcome of flipping the coin. So, the events are independent.

$$P(\text{prime}) = \frac{3}{5}$$ ← There are 3 prime numbers (2, 3, and 5). ← There is a total of 5 numbers.

$$P(\text{tails}) = \frac{1}{2}$$ ← There is 1 tails side. ← There is a total of 2 sides.

Use the formula for the probability of independent events.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$
$$P(\text{prime and tails}) = P(\text{prime}) \cdot P(\text{tails})$$
$$= \frac{3}{5} \cdot \frac{1}{2} \quad \text{Substitute.}$$
$$= \frac{3}{10} \quad \text{Multiply.}$$

∴ The probability of spinning a prime number and flipping tails is $\frac{3}{10}$, or 30%.

On Your Own

Now You're Ready
Exercises 5–8

1. What is the probability of spinning a multiple of 2 and flipping heads?

Events are **dependent events** if the occurrence of one event *does* affect the likelihood that the other event(s) will occur.

Key Idea

Probability of Dependent Events

Words The probability of two dependent events A and B is the probability of A times the probability of B after A occurs.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$

EXAMPLE 2 Finding the Probability of Dependent Events

People are randomly chosen to be game show contestants from an audience of 100 people. You are with 5 of your relatives and 6 other friends. What is the probability that one of your relatives is chosen first, and then one of your friends is chosen second?

Choosing an audience member changes the number of audience members left. So, the events are dependent.

$$P(\text{relative}) = \frac{5}{100} = \frac{1}{20}$$

There are 5 relatives.

There is a total of 100 audience members.

$$P(\text{friend}) = \frac{6}{99} = \frac{2}{33}$$

There are 6 friends.

There is a total of 99 audience members left.

Use the formula for the probability of dependent events.

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$$

$$P(\text{relative and friend}) = P(\text{relative}) \cdot P(\text{friend after relative})$$

$$= \frac{1}{20} \cdot \frac{2}{33} \qquad \text{Substitute.}$$

$$= \frac{1}{330} \qquad \text{Simplify.}$$

∴ The probability is $\frac{1}{330}$, or about 0.3%.

On Your Own

Now You're Ready
Exercises 9–12

2. What is the probability that you, your relatives, and your friends are *not* chosen to be either of the first two contestants?

EXAMPLE 3 Finding the Probability of a Compound Event

A student randomly guesses the answer for each of the multiple-choice questions. What is the probability of answering all three questions correctly?

1. In what year did the United States gain independence from Britain?

A. 1492 **B.** 1776 **C.** 1788 **D.** 1795 **E.** 2000

2. Which amendment to the Constitution grants citizenship to all persons born in the United States and guarantees them equal protection under the law?

A. 1st **B.** 5th **C.** 12th **D.** 13th **E.** 14th

3. In what year did the Boston Tea Party occur?

A. 1607 **B.** 1773 **C.** 1776 **D.** 1780 **E.** 1812

Choosing the answer for one question does not affect the choice for the other questions. So, the events are independent.

Method 1: Use the formula for the probability of independent events.

$$P(\#1 \text{ and } \#2 \text{ and } \#3 \text{ correct}) = P(\#1 \text{ correct}) \cdot P(\#2 \text{ correct}) \cdot P(\#3 \text{ correct})$$

$$= \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \quad \text{Substitute.}$$

$$= \frac{1}{125} \quad \text{Multiply.}$$

∴ The probability of answering all three questions correctly is $\frac{1}{125}$, or 0.8%.

Method 2: Use the Fundamental Counting Principle.

There are 5 choices for each question, so there are $5 \cdot 5 \cdot 5 = 125$ possible outcomes. There is only 1 way to answer all three questions correctly.

$$P(\#1 \text{ and } \#2 \text{ and } \#3 \text{ correct}) = \frac{1}{125}$$

∴ The probability of answering all three questions correctly is $\frac{1}{125}$, or 0.8%.

On Your Own

Now You're Ready
Exercises 18–22

3. The student can eliminate Choice A for all three questions. What is the probability of answering all three questions correctly? Compare this probability with the probability in Example 3. What do you notice?

10.5 Exercises

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Vocabulary and Concept Check

1. **DIFFERENT WORDS, SAME QUESTION** You randomly choose one of the chips. Without replacing the first chip, you choose a second chip. Which question is different? Find "both" answers.

   What is the probability of choosing a 1 and then a blue chip?

   What is the probability of choosing a 1 and then an even number?

   What is the probability of choosing a green chip and then a chip that is *not* red?

   What is the probability of choosing a number less than 2 and then an even number?

2. **WRITING** How do you find the probability of two events A and B when A and B are independent? dependent?

Practice and Problem Solving

Tell whether the events are *independent* or *dependent*. Explain.

3. You roll a 4 on a number cube. Then you roll an even number on a different number cube.
4. You randomly draw a lane number for a 100-meter race. Then your friend randomly draws a lane number for the same race.

You spin the spinner and flip a coin. Find the probability of the compound event.

1. 5. Spinning a 3 and flipping heads
6. Spinning an even number and flipping tails
7. Spinning a number greater than 1 and flipping tails
8. *Not* spinning a 2 and flipping heads

You randomly choose one of the tiles. Without replacing the first tile, you choose a second tile. Find the probability of the compound event.

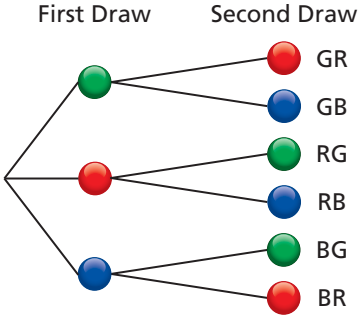
2. 9. Choosing a 5 and then a 6
10. Choosing an odd number and then a 20
11. Choosing a number less than 7 and then a multiple of 4
12. Choosing two even numbers

13. ERROR ANALYSIS Describe and correct the error in finding the probability.

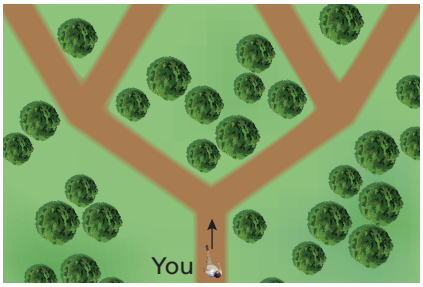
You randomly choose one of the marbles. Without replacing the first marble, you choose a second marble. What is the probability of choosing red and then green?

$P(\text{red and green}) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

14. LOGIC A bag contains three marbles. Does the tree diagram show the outcomes for *independent* or *dependent* events? Explain.



15. EARRINGS A jewelry box contains two gold hoop earrings and two silver hoop earrings. You randomly choose two earrings. What is the probability that both are silver hoop earrings?



16. HIKING You are hiking to a ranger station. There is one correct path. You come to a fork and randomly take the path on the left. You come to another fork and randomly take the path on the right. What is the probability that you are still on the correct path?

17. CARNIVAL At a carnival game, you randomly throw two darts at the board and break two balloons. What is the probability that both of the balloons you break are purple?

You spin the spinner, flip a coin, then spin the spinner again. Find the probability of the compound event.

18. Spinning a 4, flipping heads, then spinning a 7

19. Spinning an odd number, flipping heads, then spinning a 3

20. Spinning an even number, flipping tails, then spinning an odd number

21. *Not* spinning a 5, flipping heads, then spinning a 1

22. Spinning an odd number, *not* flipping heads, then *not* spinning a 6

23. LANGUAGES There are 16 students in your Spanish class. Your teacher randomly chooses one student at a time to take a verbal exam. What is the probability that you are *not* one of the first four students chosen?

24. SHOES Twenty percent of the shoes in a factory are black. One shoe is chosen and replaced. A second shoe is chosen and replaced. Then a third shoe is chosen. What is the probability that *none* of the shoes are black?

25. PROBLEM SOLVING Your teacher divides your class into two groups, and then randomly chooses a leader for each group. The probability that you are chosen to be a leader is $\frac{1}{12}$. The probability that both you and your best friend are chosen is $\frac{1}{132}$.

a. Is your best friend in your group? Explain.

b. What is the probability that your best friend is chosen as a group leader?

c. How many students are in the class?

26. Structure After ruling out some of the answer choices, you randomly guess the answer for each of the story questions below.

> **1.** Who was the oldest?
> **A.** Ned **B.** Yvonne **C.** Sun Li **D.** Angel **E.** Dusty
> **2.** What city was Stacey from?
> **A.** Raleigh **B.** New York **C.** Roanoke **D.** Dallas **E.** San Diego

a. How can the probability of getting both answers correct be 25%?

b. How can the probability of getting both answers correct be $8\frac{1}{3}\%$?

Fair Game Review What you learned in previous grades & lessons

Draw a triangle with the given angle measures. Then classify the triangle. *(Section 7.3)*

27. 30°, 60°, 90° **28.** 20°, 50°, 110° **29.** 50°, 50°, 80°

30. MULTIPLE CHOICE Which set of numbers is in order from least to greatest? *(Section 6.2)*

Ⓐ $\frac{2}{3}$, 0.6, 67% Ⓑ 44.5%, $\frac{4}{9}$, $0.4\overline{6}$

Ⓒ 0.269, 27%, $\frac{3}{11}$ Ⓓ $2\frac{1}{7}$, 214%, $2.\overline{14}$