10.3 Experimental and Theoretical Probability

Essential Question How can you use relative frequencies to find probabilities?

When you conduct an experiment, the **relative frequency** of an event is the fraction or percent of the time that the event occurs.

$$\text{relative frequency} = \frac{\text{number of times the event occurs}}{\text{total number of times you conduct the experiment}}$$

1 ACTIVITY: Finding Relative Frequencies

Work with a partner.

a. Flip a quarter 20 times and record your results. Then complete the table. Are the relative frequencies the same as the probability of flipping heads or tails? Explain.

	Flipping Heads	Flipping Tails
Relative Frequency		

b. Compare your results with those of other students in your class. Are the relative frequencies the same? If not, why do you think they differ?

c. Combine all of the results in your class. Then complete the table again. Did the relative frequencies change? What do you notice? Explain.

d. Suppose everyone in your school conducts this experiment and you combine the results. How do you think the relative frequencies will change?

2 ACTIVITY: Using Relative Frequencies

Probability and Statistics

In this lesson, you will

- find relative frequencies.
- use experimental probabilities to make predictions.
- use theoretical probabilities to find quantities.
- compare experimental and theoretical probabilities.

Work with a partner. You have a bag of colored chips. You randomly select a chip from the bag and replace it. The table shows the number of times you select each color.

Red	Blue	Green	Yellow
24	12	15	9

a. There are 20 chips in the bag. Can you use the table to find the exact number of each color in the bag? Explain.

b. You randomly select a chip from the bag and replace it. You do this 50 times, then 100 times, and you calculate the relative frequencies after each experiment. Which experiment do you think gives a better approximation of the exact number of each color in the bag? Explain.

3 ACTIVITY: Conducting an Experiment

Work with a partner. You toss a thumbtack onto a table. There are two ways the thumbtack can land.

Point up On its side

Math Practice

Analyze Relationships

How can you use the results of your experiment to determine whether this is a uniform probability model?

a. Your friend says that because there are two outcomes, the probability of the thumbtack landing point up must be $\frac{1}{2}$. Do you think this conclusion is true? Explain.

b. Toss a thumbtack onto a table 50 times and record your results. In a *uniform probability model*, each outcome is equally likely to occur. Do you think this experiment represents a uniform probability model? Explain.

Use the relative frequencies to complete the following.

$P(\text{point up}) =$ ____ $P(\text{on its side}) =$ ____

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use relative frequencies to find probabilities? Give an example.

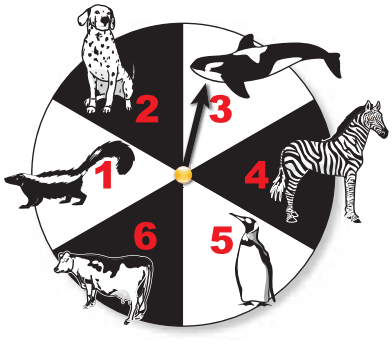
5. Your friend rolls a number cube 500 times. How many times do you think your friend will roll an odd number? Explain your reasoning.

6. In Activity 2, your friend says, "There are no orange-colored chips in the bag." Do you think this conclusion is true? Explain.

7. Give an example of an experiment that represents a uniform probability model.

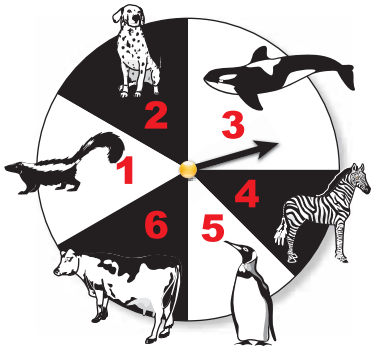
8. Tell whether you can use each spinner to represent a uniform probability model. Explain your reasoning.

a.



b.

c.



Practice Use what you learned about relative frequencies to complete Exercises 6 and 7 on page 417.

10.3 Lesson

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Key Vocabulary
relative frequency, *p. 412*
experimental probability, *p. 414*
theoretical probability, *p. 415*

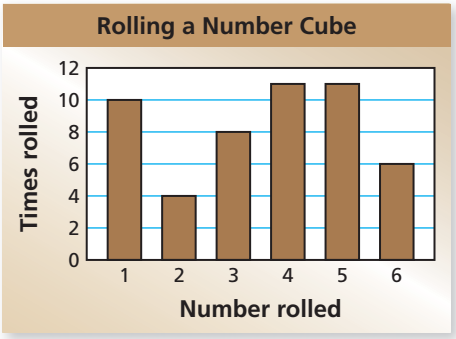
Key Idea

Experimental Probability

Probability that is based on repeated trials of an experiment is called **experimental probability**.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

EXAMPLE 1 Finding an Experimental Probability



The bar graph shows the results of rolling a number cube 50 times. What is the experimental probability of rolling an odd number?

The bar graph shows 10 ones, 8 threes, and 11 fives. So, an odd number was rolled $10 + 8 + 11 = 29$ times in a total of 50 rolls.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

$$P(\text{odd}) = \frac{29}{50}$$

An odd number was rolled 29 times.

There was a total of 50 rolls.

∴ The experimental probability is $\frac{29}{50}$, 0.58, or 58%.

EXAMPLE 2 Making a Prediction

"April showers bring May flowers." Old Proverb, 1557

It rains 2 out of the last 12 days in March. If this trend continues, how many rainy days would you expect in April?

Find the experimental probability of a rainy day.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

$$P(\text{rain}) = \frac{2}{12} = \frac{1}{6}$$

It rains 2 days.

There is a total of 12 days.

To make a prediction, multiply the probability of a rainy day by the number of days in April.

$$\frac{1}{6} \cdot 30 = 5$$

∴ So, you can predict that there will be 5 rainy days in April.

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On Your Own

Now You're Ready
Exercises 8–14

1. In Example 1, what is the experimental probability of rolling an even number?
2. At a clothing company, an inspector finds 5 defective pairs of jeans in a shipment of 200. If this trend continues, about how many pairs of jeans would you expect to be defective in a shipment of 5000?

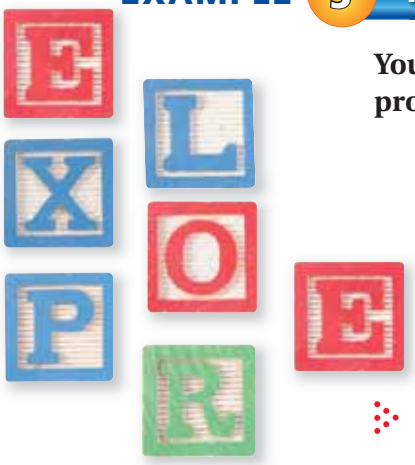
Key Idea

Theoretical Probability

When all possible outcomes are equally likely, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

EXAMPLE 3 Finding a Theoretical Probability



You randomly choose one of the letters shown. What is the theoretical probability of choosing a vowel?

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{vowel}) = \frac{3}{7}$$

← There are 3 vowels.

← There is a total of 7 letters.

∴ The probability of choosing a vowel is $\frac{3}{7}$, or about 43%.

EXAMPLE 4 Using a Theoretical Probability

The theoretical probability of winning a bobblehead when spinning a prize wheel is $\frac{1}{6}$. The wheel has 3 bobblehead sections. How many sections are on the wheel?

$$P(\text{bobblehead}) = \frac{\text{number of bobblehead sections}}{\text{total number of sections}}$$

$$\frac{1}{6} = \frac{3}{s}$$ Substitute. Let s be the total number of sections.

$$s = 18$$ Cross Products Property

∴ So, there are 18 sections on the wheel.

On Your Own

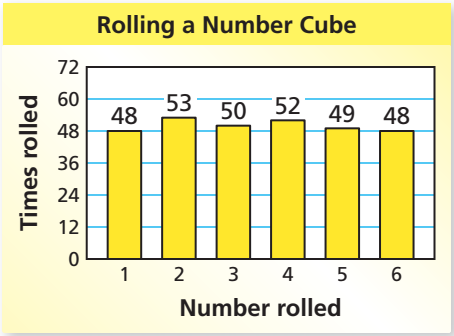
Now You're Ready
Exercises 15–23

3. In Example 3, what is the theoretical probability of choosing an X?
4. The theoretical probability of spinning an odd number on a spinner is 0.6. The spinner has 10 sections. How many sections have odd numbers?
5. The prize wheel in Example 4 was spun 540 times at a baseball game. About how many bobbleheads would you expect were won?

EXAMPLE 5 Comparing Experimental and Theoretical Probability

The bar graph shows the results of rolling a number cube 300 times.

a. What is the experimental probability of rolling an odd number?



The bar graph shows 48 ones, 50 threes, and 49 fives. So, an odd number was rolled 48 + 50 + 49 = 147 times in a total of 300 rolls.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

$$P(\text{odd}) = \frac{147}{300}$$

← An odd number was rolled 147 times.

← There was a total of 300 rolls.

$$= \frac{49}{100}, \text{ or } 49\%$$

b. How does the experimental probability compare with the theoretical probability of rolling an odd number?

In Section 10.2, Example 2, you found that the theoretical probability of rolling an odd number is 50%. The experimental probability, 49%, is close to the theoretical probability.

c. Compare the experimental probability in part (a) to the experimental probability in Example 1.

As the number of trials increased from 50 to 300, the experimental probability decreased from 58% to 49%. So, it became closer to the theoretical probability of 50%.

On Your Own

Now You're Ready
Exercises 25–27

6. Use the bar graph in Example 5 to find the experimental probability of rolling a number greater than 1. Compare the experimental probability to the theoretical probability of rolling a number greater than 1.

10.3 Exercises

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Vocabulary and Concept Check

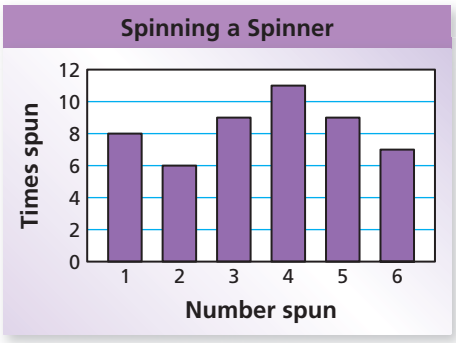
1. **VOCABULARY** Describe how to find the experimental probability of an event.
2. **REASONING** You flip a coin 10 times and find the experimental probability of flipping tails to be 0.7. Does this seem reasonable? Explain.
3. **VOCABULARY** An event has a theoretical probability of 0.5. What does this mean?
4. **OPEN-ENDED** Describe an event that has a theoretical probability of $\frac{1}{4}$.
5. **LOGIC** A pollster surveys randomly selected individuals about an upcoming election. Do you think the pollster will use experimental probability or theoretical probability to make predictions? Explain.

Practice and Problem Solving

Use the bar graph to find the relative frequency of the event.

6. Spinning a 6
7. Spinning an even number

Use the bar graph to find the experimental probability of the event.



1 8. Spinning a number less than 3
9. *Not* spinning a 1
10. Spinning a 1 or a 3
11. Spinning a 7

12. **EGGS** You check 20 cartons of eggs. Three of the cartons have at least one cracked egg. What is the experimental probability that a carton of eggs has at least one cracked egg?

2 13. **BOARD GAME** There are 105 lettered tiles in a board game. You choose the tiles shown. How many of the 105 tiles would you expect to be vowels?

14. **CARDS** You have a package of 20 assorted thank-you cards. You pick the four cards shown. How many of the 20 cards would you expect to have flowers on them?

Use the spinner to find the theoretical probability of the event.

3 **15.** Spinning red

16. Spinning a 1

17. Spinning an odd number

18. Spinning a multiple of 2

19. Spinning a number less than 7

20. Spinning a 9

21. **LETTERS** Each letter of the alphabet is printed on an index card. What is the theoretical probability of randomly choosing any letter except Z?

4 **22.** **GAME SHOW** On a game show, a contestant randomly chooses a chip from a bag that contains numbers and strikes. The theoretical probability of choosing a strike is $\frac{3}{10}$. The bag contains 9 strikes. How many chips are in the bag?

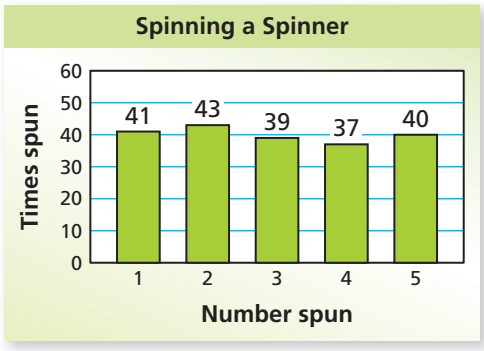
23. **MUSIC** The theoretical probability that a pop song plays on your MP3 player is 0.45. There are 80 songs on your MP3 player. How many of the songs are pop songs?

24. **MODELING** There are 16 females and 20 males in a class.

a. What is the theoretical probability that a randomly chosen student is female?

b. One week later, there are 45 students in the class. The theoretical probability that a randomly chosen student is a female is the same as last week. How many males joined the class?

The bar graph shows the results of spinning the spinner 200 times. Compare the theoretical and experimental probabilities of the event.



5 **25.** Spinning a 4

26. Spinning a 3

27. Spinning a number greater than 4

28. Should you use *theoretical* or *experimental* probability to predict the number of times you will spin a 3 in 10,000 spins?

29. **NUMBER SENSE** The table at the right shows the results of flipping two coins 12 times each.

HH	HT	TH	TT
2	6	3	1

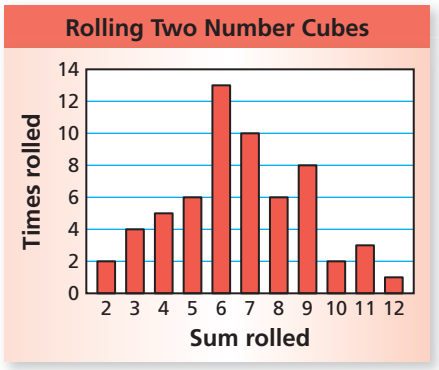
a. What is the experimental probability of flipping two tails? Using this probability, how many times can you expect to flip two tails in 600 trials?

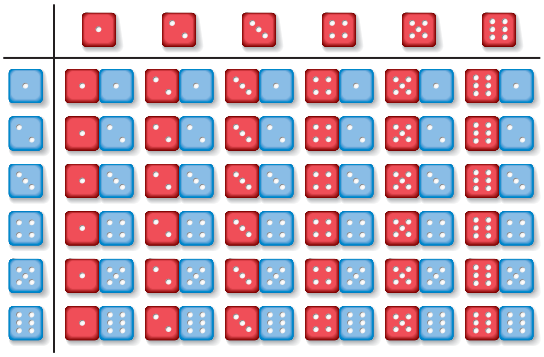
HH	HT	TH	TT
23	29	26	22

b. The table at the left shows the results of flipping the same two coins 100 times each. What is the experimental probability of flipping two tails? Using this probability, how many times can you expect to flip two tails in 600 trials?

c. Why is it important to use a large number of trials when using experimental probability to predict results?

You roll a pair of number cubes 60 times. You record your results in the bar graph shown.





30. Use the bar graph to find the experimental probability of rolling each sum. Is each sum equally likely? Explain. If not, which is most likely?

31. Use the table to find the theoretical probability of rolling each sum. Is each sum equally likely? Explain. If not, which is most likely?

32. **PROBABILITIES** Compare the probabilities you found in Exercises 30 and 31.

33. **REASONING** Consider the results of Exercises 30 and 31.

a. Which sum would you expect to be most likely after 500 trials? 1000 trials? 10,000 trials?

b. Explain how experimental probability is related to theoretical probability as the number of trials increases.

34. Project When you toss a paper cup into the air, there are three ways for the cup to land: *open-end up*, *open-end down*, or *on its side*.

a. Toss a paper cup 100 times and record your results. Do the outcomes for tossing the cup appear to be equally likely? Explain.

b. What is the probability of the cup landing open-end up? open-end down? on its side?

c. Use your results to predict the number of times the cup lands on its side in 1000 tosses.

d. Suppose you tape a quarter to the bottom of the cup. Do you think the cup will be *more likely* or *less likely* to land open-end up? Justify your answer.

Fair Game Review What you learned in previous grades & lessons

Find the annual interest rate. *(Section 6.7)*

35. $I = \$16$, $P = \$200$, $t = 2$ years

36. $I = \$26.25$, $P = \$500$, $t = 18$ months

37. **MULTIPLE CHOICE** The volume of a prism is 9 cubic yards. What is its volume in cubic feet? *(Section 9.4)*

(A) $3\ ft^3$ (B) $27\ ft^3$ (C) $81\ ft^3$ (D) $243\ ft^3$