

## 4.4 Solving Two-Step Inequalities

**Essential Question** How can you use an inequality to describe the dimensions of a figure?

### 1 ACTIVITY: Areas and Perimeters of Figures

Work with a partner.

- Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.
- Write four values of the variable that satisfy the inequality you chose.

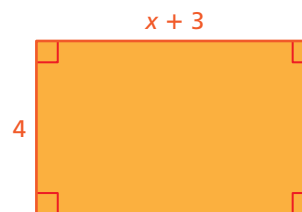
- a. You want to find the values of  $x$  so that the area of the rectangle is more than 22 square units.

$$4x + 12 > 22$$

$$4x + 3 > 22$$

$$4x + 12 \geq 22$$

$$2x + 14 > 22$$



- b. You want to find the values of  $x$  so that the perimeter of the rectangle is greater than or equal to 28 units.

$$x + 7 \geq 28$$

$$4x + 12 \geq 28$$

$$2x + 14 \geq 28$$

$$2x + 14 \leq 28$$

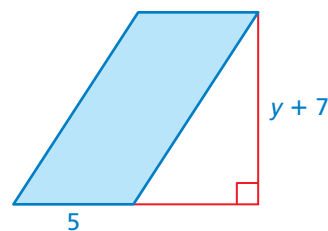
- c. You want to find the values of  $y$  so that the area of the parallelogram is fewer than 41 square units.

$$5y + 7 < 41$$

$$5y + 35 < 41$$

$$5y + 7 \leq 41$$

$$5y + 35 \leq 41$$



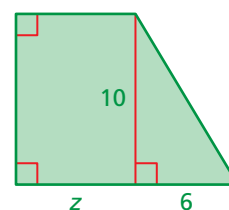
- d. You want to find the values of  $z$  so that the area of the trapezoid is at most 100 square units.

$$5z + 30 \leq 100$$

$$10z + 30 \leq 100$$

$$5z + 30 < 100$$

$$10z + 30 < 100$$



#### Inequalities

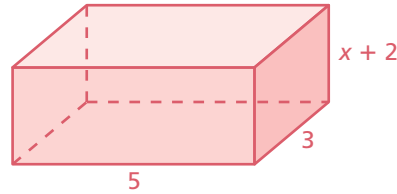
In this lesson, you will

- solve multi-step inequalities.
- solve real-life problems.

## 2 ACTIVITY: Volumes of Rectangular Prisms

Work with a partner.

- Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.
  - Write four values of the variable that satisfy the inequality you chose.
- a. You want to find the values of  $x$  so that the volume of the rectangular prism is at least 50 cubic units.



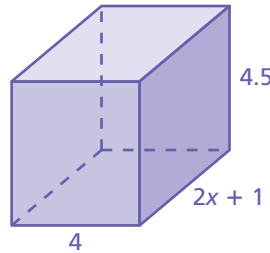
$$15x + 30 > 50$$

$$x + 10 \geq 50$$

$$15x + 30 \geq 50$$

$$15x + 2 \geq 50$$

- b. You want to find the values of  $x$  so that the volume of the rectangular prism is no more than 36 cubic units.



$$8x + 4 < 36$$

$$36x + 18 < 36$$

$$2x + 9.5 \leq 36$$

$$36x + 18 \leq 36$$

### What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use an inequality to describe the dimensions of a figure?
4. Use what you know about solving equations and inequalities to describe how you can solve a two-step inequality. Give an example to support your explanation.

### Practice

Use what you learned about solving two-step inequalities to complete Exercises 3 and 4 on page 150.

You can solve two-step inequalities in the same way you solve two-step equations.

## EXAMPLE 1 Solving Two-Step Inequalities

a. Solve  $5x - 4 \geq 11$ . Graph the solution.

Step 1: Undo the subtraction.

$$5x - 4 \geq 11$$

$$\xrightarrow{+4 \quad +4}$$

$$5x \geq 15$$

Write the inequality.

Addition Property of Inequality

Simplify.

Step 2: Undo the multiplication.

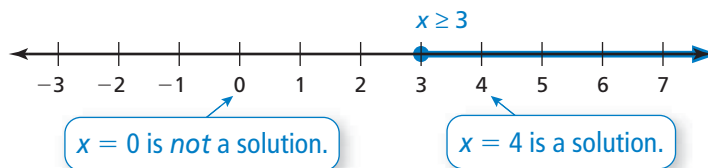
$$\xrightarrow{\frac{5x}{5} \geq \frac{15}{5}}$$

$$x \geq 3$$

Division Property of Inequality

Simplify.

∴ The solution is  $x \geq 3$ .



b. Solve  $\frac{b}{-3} + 4 < 13$ . Graph the solution.

Step 1: Undo the addition.

$$\frac{b}{-3} + 4 < 13$$

$$\xrightarrow{-4 \quad -4}$$

$$\frac{b}{-3} < 9$$

Write the inequality.

Subtraction Property of Inequality

Simplify.

Step 2: Undo the division.

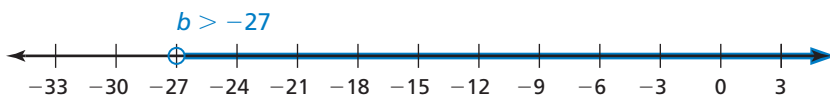
$$\xrightarrow{-3 \cdot \frac{b}{-3} > -3 \cdot 9}$$

$$b > -27$$

Use the Multiplication Property of Inequality.  
Reverse the inequality symbol.

Simplify.

∴ The solution is  $b > -27$ .



### On Your Own

Solve the inequality. Graph the solution.

Now You're Ready  
Exercises 5–10

1.  $6y - 7 > 5$

2.  $4 - 3d \geq 19$

3.  $\frac{w}{-4} + 8 > 9$

## EXAMPLE 2 Graphing an Inequality

Which graph represents the solution of  $-7(x + 3) \leq 28$ ?



$$-7(x + 3) \leq 28$$

$$-7x - 21 \leq 28$$

Step 1: Undo the subtraction.

$$\rightarrow +21 \quad +21$$

$$-7x \leq 49$$

Step 2: Undo the multiplication.

$$\rightarrow \frac{-7x}{-7} \geq \frac{49}{-7}$$

$$x \geq -7$$

Write the inequality.

Distributive Property

Addition Property of Inequality

Simplify.

Use the Division Property of Inequality.

Reverse the inequality symbol.

Simplify.

∴ The correct answer is (B).

## EXAMPLE 3 Real-Life Application

Progress Report	
Month	Pounds Lost
1	12
2	9
3	5
4	8

A contestant in a weight-loss competition wants to lose an average of at least 8 pounds per month during a 5-month period. How many pounds must the contestant lose in the fifth month to meet the goal?

Write and solve an inequality. Let  $x$  be the number of pounds lost in the fifth month.

$$\frac{12 + 9 + 5 + 8 + x}{5} \geq 8$$

The phrase *at least* means greater than or equal to.

$$\frac{34 + x}{5} \geq 8$$

Simplify.

$$5 \cdot \frac{34 + x}{5} \geq 5 \cdot 8$$

Multiplication Property of Inequality

$$34 + x \geq 40$$

Simplify.

$$x \geq 6$$

Subtract 34 from each side.

∴ So, the contestant must lose at least 6 pounds to meet the goal.

### Remember

In Example 3, the average is equal to the sum of the pounds lost divided by the number of months.

### On Your Own

Solve the inequality. Graph the solution.

4.  $2(k - 5) < 6$

5.  $-4(n - 10) < 32$

6.  $-3 \leq 0.5(8 + y)$

7. **WHAT IF?** In Example 3, the contestant wants to lose an average of at least 9 pounds per month. How many pounds must the contestant lose in the fifth month to meet the goal?

Now You're Ready  
Exercises 12–17

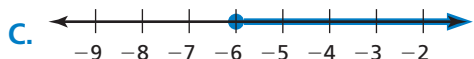
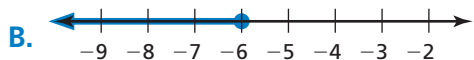
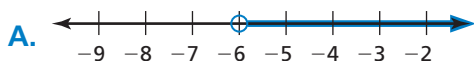
## Vocabulary and Concept Check

- WRITING** Compare and contrast solving two-step inequalities and solving two-step equations.
- OPEN-ENDED** Describe how to solve the inequality  $3(a + 5) < 9$ .

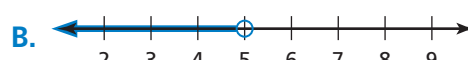
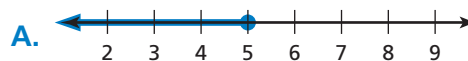
## Practice and Problem Solving

Match the inequality with its graph.

3.  $\frac{t}{3} - 1 \geq -3$



4.  $5x + 7 \leq 32$



Solve the inequality. Graph the solution.

1 5.  $8y - 5 < 3$

6.  $3p + 2 \geq -10$

7.  $2 > 8 - \frac{4}{3}h$

8.  $-2 > \frac{m}{6} - 7$

9.  $-1.2b - 5.3 \geq 1.9$

10.  $-1.3 \geq 2.9 - 0.6r$

11. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

**X**  $\frac{x}{3} + 4 < 6$   
 $x + 4 < 18$   
 $x < 14$

Solve the inequality. Graph the solution.

2 12.  $5(g + 4) > 15$

13.  $4(w - 6) \leq -12$

14.  $-8 \leq \frac{2}{5}(k - 2)$

15.  $-\frac{1}{4}(d + 1) < 2$

16.  $7.2 > 0.9(n + 8.6)$

17.  $20 \geq -3.2(c - 4.3)$



18. **UNICYCLE** The first jump in a unicycle high-jump contest is shown. The bar is raised 2 centimeters after each jump. Solve the inequality  $2n + 10 \geq 26$  to find the number of additional jumps needed to meet or exceed the goal of clearing a height of 26 centimeters.

**Solve the inequality. Graph the solution.**

19.  $9x - 4x + 4 \geq 36 - 12$

20.  $3d - 7d + 2.8 < 5.8 - 27$

21. **SCUBA DIVER** A scuba diver is at an elevation of  $-38$  feet. The diver starts moving at a rate of  $-12$  feet per minute. Write and solve an inequality that represents how long it will take the diver to reach an elevation deeper than  $-200$  feet.

22. **KILLER WHALES** A killer whale has eaten 75 pounds of fish today. It needs to eat at least 140 pounds of fish each day.

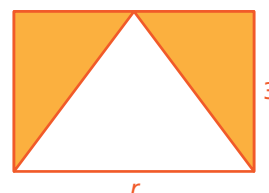
- a. A bucket holds 15 pounds of fish. Write and solve an inequality that represents how many more buckets of fish the whale needs to eat.
- b. Should the whale eat *four* or *five* more buckets of fish? Explain.



23. **REASONING** A student theater charges \$9.50 per ticket.

- a. The theater has already sold 70 tickets. Write and solve an inequality that represents how many more tickets the theater needs to sell to earn at least \$1000.
- b. The theater increases the ticket price by \$1. Without solving an inequality, describe how this affects the total number of tickets needed to earn at least \$1000.

24. **Problem Solving** For what values of  $r$  will the area of the shaded region be greater than or equal to 12 square units?



**Fair Game Review** what you learned in previous grades & lessons

**Find the missing values in the ratio table. Then write the equivalent ratios.**

*(Skills Review Handbook)*

25.

Flutes	7		28
Clarinets	4	12	

26.

Boys	6	3	
Girls	10		50

27. **MULTIPLE CHOICE** What is the volume of the cube?

*(Skills Review Handbook)*

- (A)  $8 \text{ ft}^3$
- (B)  $16 \text{ ft}^3$
- (C)  $24 \text{ ft}^3$
- (D)  $32 \text{ ft}^3$

