2.4 Multiplying and Dividing Rational Numbers

Essential Question Why is the product of two negative rational numbers positive?

In Section 1.4, you used a table to see that the product of two negative integers is a positive integer. In this activity, you will find that same result another way.

1 ACTIVITY: Showing $(-1)(-1) = 1$

Work with a partner. How can you show that $(-1)(-1) = 1$?

From the Additive Inverse Property, you know that $1 + (-1) = 0$. If you can show that $(-1)(-1) + (-1) = 0$ is true, then you have shown that $(-1)(-1) = 1$.

Justify each step.

$$(-1)(-1) + (-1) = (-1)(-1) + 1(-1)$$

$$= (-1)[(-1) + 1]$$

$$= (-1)0$$

$$= 0$$

So, $(-1)(-1) = 1$.

2 ACTIVITY: Multiplying by -1

Work with a partner.

a. Graph each number below on three different number lines. Then multiply each number by -1 and graph the product on the appropriate number line.

$$2 \qquad\qquad 8 \qquad\qquad -1$$

Rational Numbers

In this lesson, you will
- multiply and divide rational numbers.
- solve real-life problems.

b. How does multiplying by -1 change the location of the points in part (a)? What is the relationship between the number and the product?

c. Graph each number below on three different number lines. Where do you think the points will be after multiplying by -1? Plot the points. Explain your reasoning.

$$\frac{1}{2} \qquad\qquad 2.5 \qquad\qquad -\frac{5}{2}$$

d. What is the relationship between a rational number $-a$ and the product $-1(a)$? Explain your reasoning.

3 ACTIVITY: Understanding the Product of Rational Numbers

Work with a partner. Let a and b be positive rational numbers.

a. Because a and b are positive, what do you know about $-a$ and $-b$?

b. Justify each step.

$$(-a)(-b) = (-1)(a)(-1)(b)$$

$$= (-1)(-1)(a)(b)$$

$$= (1)(a)(b)$$

$$= ab$$

c. Because a and b are positive, what do you know about the product ab?

d. What does this tell you about products of rational numbers? Explain.

4 ACTIVITY: Writing a Story

Work with a partner. Write a story that uses addition, subtraction, multiplication, or division of rational numbers.

- At least one of the numbers in the story has to be negative and *not* an integer.
- Draw pictures to help illustrate what is happening in the story.
- Include the solution of the problem in the story.

Math Practice

Specify Units
What units are in your story?

If you are having trouble thinking of a story, here are some common uses of negative numbers:

- A profit of $-\$15$ is a loss of \$15.
- An elevation of -100 feet is a depth of 100 feet below sea level.
- A gain of -5 yards in football is a loss of 5 yards.
- A score of -4 in golf is 4 strokes under par.

What Is Your Answer?

5. IN YOUR OWN WORDS Why is the product of two negative rational numbers positive?

6. PRECISION Show that $(-2)(-3) = 6$.

7. How can you show that the product of a negative rational number and a positive rational number is negative?

Practice ➤ Use what you learned about multiplying rational numbers to complete Exercises 7–9 on page 68.

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Key Idea

Multiplying and Dividing Rational Numbers

Words To multiply or divide rational numbers, use the same rules for signs as you used for integers.

Remember

The *reciprocal* of $\frac{a}{b}$ is $\frac{b}{a}$.

Numbers
$$-\frac{2}{7} \cdot \frac{1}{3} = \frac{-2 \cdot 1}{7 \cdot 3} = \frac{-2}{21} = -\frac{2}{21}$$

$$-\frac{1}{2} \div \frac{4}{9} = \frac{-1}{2} \cdot \frac{9}{4} = \frac{-1 \cdot 9}{2 \cdot 4} = \frac{-9}{8} = -\frac{9}{8}$$

EXAMPLE 1 **Dividing Rational Numbers**

Find $-5\frac{1}{5} \div 2\frac{1}{3}$.

Estimate $-5 \div 2 = -2\frac{1}{2}$

$$-5\frac{1}{5} \div 2\frac{1}{3} = -\frac{26}{5} \div \frac{7}{3}$$ Write mixed numbers as improper fractions.

$$= \frac{-26}{5} \cdot \frac{3}{7}$$ Multiply by the reciprocal of $\frac{7}{3}$.

$$= \frac{-26 \cdot 3}{5 \cdot 7}$$ Multiply the numerators and the denominators.

$$= \frac{-78}{35}, \text{ or } -2\frac{8}{35}$$ Simplify.

The quotient is $-2\frac{8}{35}$.   **Reasonable?** $-2\frac{8}{35} \approx -2\frac{1}{2}$ ✓

EXAMPLE 2 **Multiplying Rational Numbers**

Find $-2.5 \cdot 3.6$.

$$\begin{array}{r} -2.5 \\ \times\ 3.6 \\ \hline 1\ 5\ 0 \\ 7\ 5\ 0 \\ \hline -9.0\ 0 \end{array}$$

The decimals have different signs.

The product is negative.

The product is -9.

EXAMPLE 3

Multiplying More Than Two Rational Numbers

Find $-\dfrac{1}{7} \cdot \left[\dfrac{4}{5} \cdot (-7)\right].$

You can use properties of multiplication to make the product easier to find.

$$-\dfrac{1}{7} \cdot \left[\dfrac{4}{5} \cdot (-7)\right] = -\dfrac{1}{7} \cdot \left(-7 \cdot \dfrac{4}{5}\right) \qquad \text{Commutative Property of Multiplication}$$

$$= -\dfrac{1}{7} \cdot (-7) \cdot \dfrac{4}{5} \qquad \text{Associative Property of Multiplication}$$

$$= 1 \cdot \dfrac{4}{5} \qquad \text{Multiplicative Inverse Property}$$

$$= \dfrac{4}{5} \qquad \text{Multiplication Property of One}$$

⋮ The product is $\dfrac{4}{5}$.

On Your Own

Now You're Ready
Exercises 10–30

Multiply or divide. Write fractions in simplest form.

1. $-\dfrac{6}{5} \div \left(-\dfrac{1}{2}\right)$

2. $\dfrac{1}{3} \div \left(-2\dfrac{2}{3}\right)$

3. $1.8(-5.1)$

4. $-6.3(-0.6)$

5. $-\dfrac{2}{3} \cdot 7\dfrac{7}{8} \cdot \dfrac{3}{2}$

6. $-7.2 \cdot 0.1 \cdot (-100)$

EXAMPLE 4 Real-Life Application

Account Positions	⟳		
Stock	Original Value	Current Value	Change
A	600.54	420.15	−180.39
B	391.10	518.38	127.28
C	380.22	99.70	−280.52

An investor owns Stocks A, B, and C. What is the mean change in the value of the stocks?

$$\text{mean} = \dfrac{-180.39 + 127.28 + (-280.52)}{3} = \dfrac{-333.63}{3} = -111.21$$

⋮ The mean change in the value of the stocks is −\$111.21.

On Your Own

7. **WHAT IF?** The change in the value of Stock D is \$568.23. What is the mean change in the value of the four stocks?

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✓ Vocabulary and Concept Check

1. **WRITING** How is multiplying and dividing rational numbers similar to multiplying and dividing integers?

2. **NUMBER SENSE** Find the reciprocal of $-\dfrac{2}{5}$.

Tell whether the expression is *positive* or *negative* without evaluating.

3. $-\dfrac{3}{10} \times \left(-\dfrac{8}{15}\right)$ 4. $1\dfrac{1}{2} \div \left(-\dfrac{1}{4}\right)$ 5. -6.2×8.18 6. $\dfrac{-8.16}{-2.72}$

📝 Practice and Problem Solving

Multiply.

7. $-1\left(\dfrac{4}{5}\right)$ 8. $-1\left(-3\dfrac{1}{2}\right)$ 9. $-0.25(-1)$

Divide. Write fractions in simplest form.

① 10. $-\dfrac{7}{10} \div \dfrac{2}{5}$ 11. $\dfrac{1}{4} \div \left(-\dfrac{3}{8}\right)$ 12. $-\dfrac{8}{9} \div \left(-\dfrac{8}{9}\right)$ 13. $-\dfrac{1}{5} \div 20$

14. $-2\dfrac{4}{5} \div (-7)$ 15. $-10\dfrac{2}{7} \div \left(-4\dfrac{4}{11}\right)$ 16. $-9 \div 7.2$ 17. $8 \div 2.2$

18. $-3.45 \div (-15)$ 19. $-0.18 \div 0.03$ 20. $8.722 \div (-3.56)$ 21. $12.42 \div (-4.8)$

Multiply. Write fractions in simplest form.

② ③ 22. $-\dfrac{1}{4} \times \left(-\dfrac{4}{3}\right)$ 23. $\dfrac{5}{6}\left(-\dfrac{8}{15}\right)$ 24. $-2\left(-1\dfrac{1}{4}\right)$

25. $-3\dfrac{1}{3} \cdot \left(-2\dfrac{7}{10}\right)$ 26. $0.4 \times (-0.03)$ 27. $-0.05 \times (-0.5)$

28. $-8(0.09)(-0.5)$ 29. $\dfrac{5}{6} \cdot \left(-4\dfrac{1}{2}\right) \cdot \left(-2\dfrac{1}{5}\right)$ 30. $\left(-1\dfrac{2}{3}\right)^3$
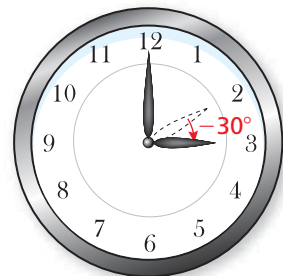
ERROR ANALYSIS Describe and correct the error.

31.
✗ $-2.2 \times 3.7 = 8.14$

32.
✗ $-\dfrac{1}{4} \div \dfrac{3}{2} = -\dfrac{4}{1} \times \dfrac{3}{2} = -\dfrac{12}{2} = -6$

33. **HOUR HAND** The hour hand of a clock moves $-30°$ every hour. How many degrees does it move in $2\dfrac{1}{5}$ hours?

34. **SUNFLOWER SEEDS** How many 0.75-pound packages can you make with 6 pounds of sunflower seeds?

Evaluate.

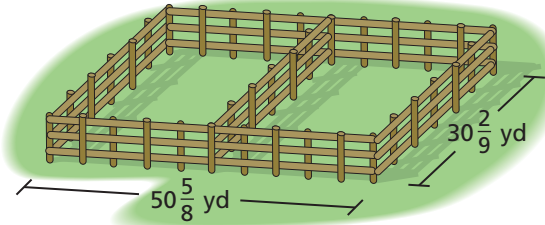**35.** $-4.2 + 8.1 \times (-1.9)$

36. $2.85 - 6.2 \div 2^2$

37. $-3.64 \cdot \left| -5.3 \right| - 1.5^3$

38. $1\frac{5}{9} \div \left(-\frac{2}{3}\right) + \left(-2\frac{3}{5}\right)$

39. $-3\frac{3}{4} \times \frac{5}{6} - 2\frac{1}{3}$

40. $\left(-\frac{2}{3}\right)^2 - \frac{3}{4}\left(2\frac{1}{3}\right)$

41. OPEN-ENDED Write two fractions whose product is $-\frac{3}{5}$.

42. FENCING A farmer needs to enclose two adjacent rectangular pastures. How much fencing does the farmer need?

$30\frac{2}{9}$ yd

$50\frac{5}{8}$ yd

43. GASOLINE A 14.5-gallon gasoline tank is $\frac{3}{4}$ full. How many gallons will it take to fill the tank?

44. PRECISION A section of a boardwalk is made using 15 boards. Each board is $9\frac{1}{4}$ inches wide. The total width of the section is 144 inches. The spacing between each board is equal. What is the width of the spacing between each board?

45. RUNNING The table shows the changes in the times (in seconds) of four teammates. What is the mean change?

Teammate	Change
1	-2.43
2	-1.85
3	0.61
4	-1.45

46. Critical Thinking The daily changes in the barometric pressure for four days are -0.05, 0.09, -0.04, and -0.08 inches.

  a. What is the mean change?

  b. The mean change after five days is -0.01 inch. What is the change on the fifth day? Explain.

Fair Game Review *What you learned in previous grades & lessons*

Add or subtract. *(Section 2.2 and Section 2.3)*

47. $-6.2 + 4.7$

48. $-8.1 - (-2.7)$

49. $\frac{9}{5} - \left(-2\frac{7}{10}\right)$

50. $-4\frac{5}{6} + \left(-3\frac{4}{9}\right)$

51. MULTIPLE CHOICE What are the coordinates of the point in Quadrant IV? *(Skills Review Handbook)*

  A $(-4, 1)$

  B $(-3, -3)$

  C $(0, -2)$

  D $(3, -3)$