4.1 Writing and Graphing Inequalities

Essential Question How can you use a number line to represent solutions of an inequality?

1 ACTIVITY: Understanding Inequality Statements

Work with a partner. Read the statement. Circle each number that makes the statement true, and then answer the questions.

a. "You are in at least 5 of the photos."

−3 −2 −1 0 1 2 3 4 5 6

- What do you notice about the numbers that you circled?
- Is the number 5 included? Why or why not?
- Write four other numbers that make the statement true.

b. "The temperature is less than −4 degrees Fahrenheit."

−7 −6 −5 −4 −3 −2 −1 0 1 2

- What do you notice about the numbers that you circled?
- Can the temperature be exactly −4 degrees Fahrenheit? Explain.
- Write four other numbers that make the statement true.

c. "More than 3 students from our school are in the chess tournament."

−3 −2 −1 0 1 2 3 4 5 6

- What do you notice about the numbers that you circled?
- Is the number 3 included? Why or why not?
- Write four other numbers that make the statement true.

d. "The balance in a yearbook fund is no more than −$5."

−7 −6 −5 −4 −3 −2 −1 0 1 2

- What do you notice about the numbers that you circled?
- Is the number −5 included? Why or why not?
- Write four other numbers that make the statement true.

Inequalities

In this lesson, you will

- write and graph inequalities.
- use substitution to check whether a number is a solution of an inequality.

2 ACTIVITY: Understanding Inequality Symbols

Work with a partner.

a. Consider the statement "x is a number such that $x > -1.5$."

- Can the number be exactly -1.5? Explain.
- Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
- Write four other numbers that are not integers that make the statement true.

b. Consider the statement "x is a number such that $x \leq \frac{5}{2}$."

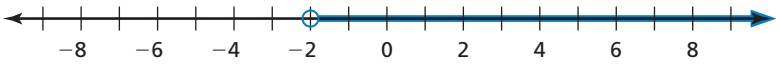
- Can the number be exactly $\frac{5}{2}$? Explain.
- Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
- Write four other numbers that are not integers that make the statement true.

3 ACTIVITY: Writing and Graphing Inequalities

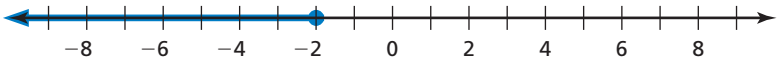
Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of x that make the inequality true.

a.

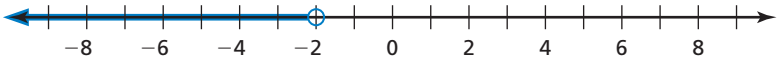
b.



c.



d.



Math Practice

Check Progress

All the graphs are similar. So, what can you do to make sure that you have correctly written each inequality?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a number line to represent solutions of an inequality?

5. **STRUCTURE** Is $x \geq -1.4$ the same as $-1.4 \leq x$? Explain.

Practice Use what you learned about writing and graphing inequalities to complete Exercises 4 and 5 on page 128.

4.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
inequality, *p. 126*
solution of an inequality, *p. 126*
solution set, *p. 126*
graph of an inequality, *p. 127*

An **inequality** is a mathematical sentence that compares expressions. It contains the symbols $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine where to place the inequality symbol.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 Writing an Inequality

A number q plus 5 is greater than or equal to -7.9. Write this word sentence as an inequality.

A number q plus 5 is greater than or equal to -7.9.
$q + 5$ $\geq$ -7.9

∴ An inequality is $q + 5 \geq -7.9$.

On Your Own

Now You're Ready
Exercises 6–9

Write the word sentence as an inequality.

1. A number x is at most -10.
2. Twice a number y is more than $-\frac{5}{2}$.

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 2 \leq -1$	Is the inequality true?
-2	$-2 + 2 \stackrel{?}{\leq} -1$ $0 \not\leq -1$ ✗	no
-3	$-3 + 2 \stackrel{?}{\leq} -1$ $-1 \leq -1$ ✓	yes
-4	$-4 + 2 \stackrel{?}{\leq} -1$ $-2 \leq -1$ ✓	yes

Reading
The symbol $\not\leq$ means *is not less than or equal to*.

EXAMPLE 2 Checking Solutions

Tell whether −2 is a solution of each inequality.

a. $y - 5 \geq -6$

$y - 5 \geq -6$ Write the inequality.

$-2 - 5 \stackrel{?}{\geq} -6$ Substitute -2 for y.

$-7 \not\geq -6$ ✗ Simplify.

-7 is *not* greater than or equal to -6.

∴ So, -2 is *not* a solution of the inequality.

b. $-5.5y < 14$

$-5.5y < 14$

$-5.5(-2) \stackrel{?}{<} 14$

$11 < 14$ ✓

11 is less than 14.

∴ So, -2 is a solution of the inequality.

On Your Own

Now You're Ready
Exercises 11–16

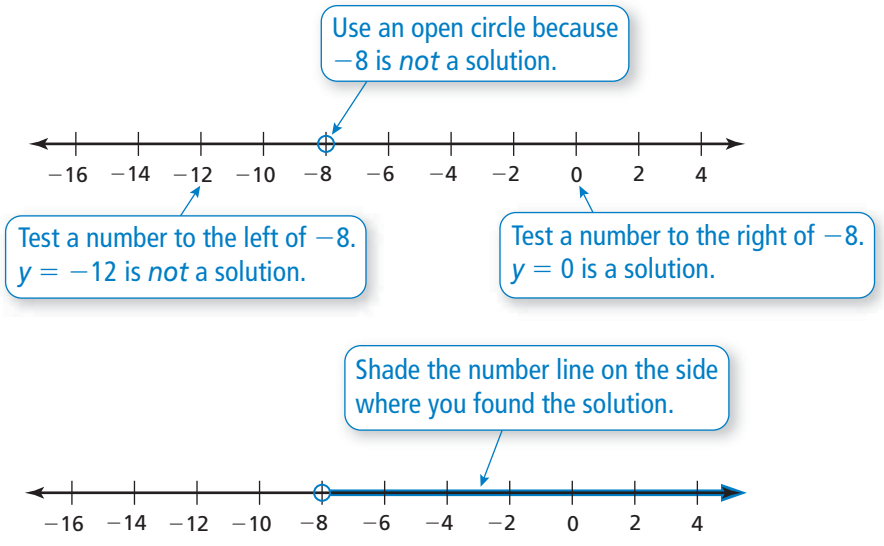
Tell whether −5 is a solution of the inequality.

3. $x + 12 > 7$ **4.** $1 - 2p \leq -9$ **5.** $n \div 2.5 \geq -3$

The **graph of an inequality** shows all the solutions of the inequality on a number line. An open circle ○ is used when a number is *not* a solution. A closed circle ● is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 Graphing an Inequality

Graph $y > -8$.



Study Tip

The graph in Example 3 shows that the inequality has *infinitely many* solutions.

On Your Own

Now You're Ready
Exercises 17–20

Graph the inequality on a number line.

6. $x < -1$ **7.** $z \geq 4$ **8.** $s \leq 1.4$ **9.** $-\frac{1}{2} < t$

4.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **PRECISION** Should you use an open circle or a closed circle in the graph of the inequality $b \geq -42$? Explain.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

   k is less than or equal to -3.

   k is no more than -3.

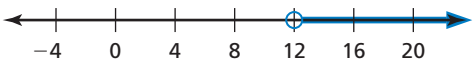
   k is at most -3.

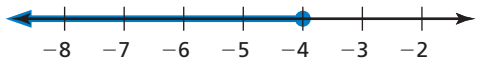
   k is at least -3.

3. **REASONING** Do $x < 5$ and $5 < x$ represent the same inequality? Explain.

Practice and Problem Solving

Write an inequality for the graph. Then, in words, describe all the values of x that make the inequality true.

4. 

5. 

Write the word sentence as an inequality.

1 6. A number y is no more than -8.

7. A number w added to 2.3 is more than 18.

8. A number t multiplied by -4 is at least $-\frac{2}{5}$.

9. A number b minus 4.2 is less than -7.5.

10. **ERROR ANALYSIS** Describe and correct the error in writing the word sentence as an inequality.

> ✗ Twice a number x is at most −24.
>
> $2x \geq -24$

Tell whether the given value is a solution of the inequality.

2 11. $n + 8 \leq 13; n = 4$

12. $5h > -15; h = -5$

13. $p + 1.4 \leq 0.5; p = 0.1$

14. $\frac{a}{6} > -4; a = -18$

15. $-\frac{2}{3}s \geq 6; s = -9$

16. $\frac{7}{8} - 3k < -\frac{1}{2}; k = \frac{1}{4}$

Graph the inequality on a number line.

3 17. $r \leq -9$

18. $g > 2.75$

19. $x \geq -3\frac{1}{2}$

20. $z < 1\frac{1}{4}$

21. **FOOD TRUCK** Each day at lunchtime, at least 53 people buy food from a food truck. Write an inequality that represents this situation.

Tell whether the given value is a solution of the inequality.

22. $4k < k + 8; k = 3$

23. $\frac{w}{3} \geq w - 12; w = 15$

24. $7 - 2y > 3y + 13; y = -1$

25. $\frac{3}{4}b - 2 \leq 2b + 8; b = -4$

26. **MODELING** A subway ride for a student costs \$1.25. A monthly pass costs \$35.

a. Write an inequality that represents the number of times you must ride the subway for the monthly pass to be a better deal.

b. You ride the subway about 45 times per month. Should you buy the monthly pass? Explain.

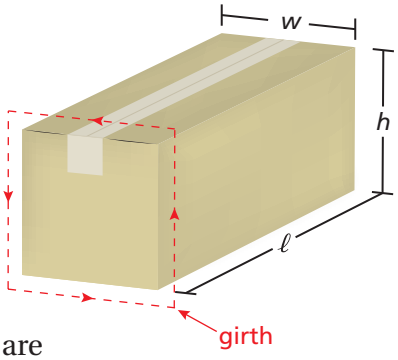
27. **LOGIC** Consider the inequality $b > -2$.

a. Describe the values of b that are solutions of the inequality.

b. Describe the values of b that are *not* solutions of the inequality. Write an inequality for these values.

c. What do all the values in parts (a) and (b) represent? Is this true for any inequality?

28. Critical Thinking A postal service says that a rectangular package can have a maximum combined length and *girth* of 108 inches. The girth of a package is the distance around the perimeter of a face that does not include the length.



a. Write an inequality that represents the allowable dimensions for the package.

b. Find three different sets of allowable dimensions that are reasonable for the package. Find the volume of each package.

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 3.3)*

29. $p - 8 = 3$ **30.** $8.7 + w = 5.1$ **31.** $x - 2 = -9$

32. **MULTIPLE CHOICE** Which expression has a value less than -5? *(Section 1.2)*

(A) $5 + 8$ (B) $-9 + 5$ (C) $1 + (-8)$ (D) $7 + (-2)$